Goddard

NO

SURVIVAL OF COMET NUCLEI AND THE ASTEROIDS*

Ernst J. Öpik†
Department of Physics
University of Maryland
College Park, Maryland

Abstract

Formulae for the probability of collision and orbital change in close encounters with the planets are further developed, auxiliary tables calculated, and the theory applied to comets and asteroidal populations. Probabilities and lifetimes for collision and ultimate orbital change are calculated for selected objects, or groups of objects, crossing the orbits of the principal planets.

Objects crossing Jupiter's orbit are chiefly eliminated by orbital change and ejection to infinity, and have a relatively short lifetime of the order of 10^6 years. Those confined to the region of the terrestrial planets are chiefly removed by physical collisions and are more longlived, with a lifetime of the order of 10^8 years for the Apollo group, and 6×10^9 years for those crossing the orbit of Mars alone, so that about 50 per cent of their original population may have survived from the beginnings of the solar system. On the contrary, the Apollo group objects cannot have survived over so long intervals of time; their present number must depend upon the balance between elimination and supply from other sources.

The origin, structure and dimensions of comet nuclei are reviewed. Oort's hypothesis of their origin as asteroids ejected from the inner

* Supported by a grant from the National Aeronautics and Space Administration Grant NsG-58-60. Paper presented at the Comet Symposium of the American Astronomical Society at Nantucket, June 20, 1961.

† On leave from Armagh Observatory, Northern Ireland.

portions of the solar system is considered as most plausible. Their re-capture by Jupiter, and by the terrestrial planets as a second stage, feeds comets into "terrestrial" space, or that inside Jupiter's orbit. However, the chief process in decreasing their aphelion distances appears to be Whipple's "rocket effect" for nuclei in retrograde rotation.

Statistical grounds are pointing to Apollo group asteroids chiefly originating from residual nuclei of comets, the supply from the asteroidal belt by Mars perturbations being inadequate though not negligible.

A formula for estimating the diameters of comet nuclei from photometric data is proposed and statistically checked as to order of magnitude. The average diameter of the nucleus of a comet of the sixth absolute magnitude (specially defined) is estimated to be 10 km, an order of magnitude smaller than suggested by former estimates. The new estimates are also supported by the apparent decrease of the gravitational constant, caused by the jet of gases ejected sunward.

1. INTRODUCTION

Comets represent the most conspicuous group of changing objects in the solar system. Their orbits, affected by stellar perturbations at great distances, and by planetary perturbations when near the Sun, are intrinsically unstable and, by shedding off matter, they are undergoing a process of disintegration which may end either in complete or in partial destruction. The changes in comets are directly observable, involving processes which may have been instrumental in the formation of the members of the solar system. With the cosmogonic implications in mind, this article considers in particular the dynamical survival of comets and other stray bodies in encounters with the planets; hence also a clue to a possible genetic link between comets and asteroids is indicated statistically.

2. SURVIVAL IN ENCOUNTERS.

(a) The statistical setting

Two intersecting orbits may result in large perturbations at close encounters, or even in a physical collision. The same holds for crossing orbits which do not intersect but which cover a common range in heliocentric distance; this follows from the general character of the secular perturbations, usually a precession of the node and advance of the perihelion which, after intervals of the order of $10^4 - 10^5$ years in the inner portions of the solar system, lead repeatedly to intersection. An orbit of a small body crossing that of a principal planet is therefore intrinsically unstable, unless a particular mechanism of commensurability (e.g., in the case of Pluto with respect to Neptune[1]) prevents the two objects from coming close together. For comets and most other stray bodies no such commensurability exists, so that their survival is a matter of statistical expectation.

The expectation of orbital change of objects crossing the orbits of planets can be treated according to the theory of probabilities. The fate of individual bodies may be virtually unpredictable by the methods of celestial mechanics, but statistical predictions may still hold as averages for an entire population.

Only interactions at close encounters are considered here; these lead to the major changes. Orbital change from perturbations at great distances is less drastic and, in comparison, can be disregarded.

The probabilities depend on the orbital elements and, as these are changed in an unpredictable manner, the problem becomes highly involved when individual objects are considered. However, if a steady or slowly changing orbital population of the stray bodies (comets) is assumed, calculations based on any existing sets of orbital elements will yield correct average results.

For a similar reason, mathematical simplifications can be introduced without essentially affecting the calculated probabilities. The mass of the stray body is assumed to be infinitesimal, the orbit of the planet a circle as a first approximation (its eccentricity being allowed for as a second step), and the motion of a stray body inside D, a conventionally defined radius of the sphere of action, is assumed to be governed only by the gravitational field of the planet, and outside D only by the solar field. D is defined by the condition that the solar perturbation on the body when in conjunction with the sun be equal to the planet's attraction[2] and, in units of the planet's orbital radius, equals

$$D = \left(\tfrac{1}{2}\mu\right)^{1/3} \qquad (1)$$

where μ is the mass of the planet in solar units, assumed to be small. Tisserand's definition is somewhat different, but not essentially and less suitable for our purposes.

In such a manner the problem of encounter is reduced to a combination of two two-body problems.

(b) Probability of close passage and collision

The relevant formulae for physical collision and cumulative angular deflection in repeated encounters have been given by the author[2](1). With the planet's mean heliocentric distance as unit of length, and setting equal to unity the sun's mass, the planet's mean orbital velocity and mean motion (unit of time = $1/2\pi$ of the period of revolution; gravitational constant = 1), in a frame of cartesian coordinates rotating with the "mean planet", near the point of intersection of the two orbits but outside the sphere of action of the planet, the components of the unperturbed velocity of the stray body (further also called "the particle") relative to the mean planet are

$$U_x^2 = 2 - A(1 - e^2) - A^{-1} \tag{2}$$

(x-axis in the radial direction toward the sun),

$$U_y = + \left[A(1-e^2)\right]^{1/2} \cos i - 1 \tag{3}$$

(y-axis tangential in the direction of the planet's orbital motion), and

$$U_z^2 = A(1-e^2) \sin^2 i \tag{4}$$

The total relative velocity U, an invariant in repeated encounters within the frame of our mathematical simplifications, is then

$$U^2 = 3 - 2\left[A(1-e^2)\right]^{1/2} \cos i - A^{-1} \tag{5}$$

Here A, e, and i are the semi-major axis, eccentricity, and inclination (relative to the planet) of the heliocentric orbit of the particle. The "mean planet" is defined as one moving in a circular orbit at the mean distance of the real planet.

Unless perturbed by a planet, in purely Keplerian motion around the sun the U-components remain unchanged. In close encounters with the planets, the components may change and the U-vector change direction, its absolute value remaining constant.

Let s denote the target radius at encounter ("impact parameter") or the distance of the asymptote of the hyperbolic relative orbit of the particle from the planet, in the conventionally assumed two-body interaction during the passage of the particle through the sphere of action of the planet. The mathematical expectation, p (to be further called "probability") of a passage within target radius s from the planet is

$$p = \frac{f\, s^2}{\pi \sin i} \left(\frac{U^2 + 0.44\, e_0^2}{U_x^2 + 0.44\, e_0^2} \right)^{1/2} \tag{6}$$

per heliocentric revolution of the particle. Here e_0 is the eccentricity of the planet's orbit (averaged over secular perturbations) and f the "overlapping fraction" or the fraction of the planet's orbit over which crossing is possible. Usually the crossing is complete and $f = 1$. For a single passage the actual inclination is to be taken, but the formula does not apply when i is very small ($< 0\overset{\circ}{.}4$). Over long intervals of time and when $\sin i$ is small, the statistical average defined through

$$\sin^2 i = \sin^2 i_c + \sin^2 i_o \qquad (7)$$

can be used; here i_c and i_o are the average inclinations, relative to the invariable plane, of the orbits of particle and planet, respectively.

The relation of periastron (perigee) distance, r, to the target radius, s, in the two-body encounter of particle with planet is

$$s = r\,(1 + 2\mu/r\,U^2)^{1/2} \qquad (8)$$

Substituting this into equation (6), an expression for the probability of a planetocentric passage, at a distance closer or equal to r, can be obtained.

For nearly parabolic, non-periodic orbits, p is the average probability per apparition for objects with similar orbital elements if the longitudes of their nodes and perihelia are distributed at random; the parameter, $A(1-e^2)$, is then to be set equal to $2Q$, the double perihelion distance.

For a periodic orbit, of a period $(a_o A)^{1.5}$ in years, where a_o is the mean heliocentric distance of the planet in astronomical units, the probability P per year, corresponding to a reciprocal of the lifetime τ, is

$$1/\tau = P = p\,(a_o\,A)^{-1.5}\ (yr^{-1}) \qquad (9)$$

When $r = R$, the radius of the planet, equations (6), (9), and (8) define the probability of physical collision and its target radius $s = S$.

(c) Probability of angular deflection

In the conventionally assumed two-body free encounter, when $r > R$, the U-vector remains constant and only changes direction by an angle γ, given by

$$\sin {}^1\!/_2\,\gamma = (1 + rU^2/\mu)^{-1} \qquad (10)$$

or

$$\tan(45^{\circ} + {}^{1}/_{4}\gamma) = (1 + 2\mu/rU^2)^{1/2} = s/r \qquad (11)$$

The direction of the U-vector determines the elements of the heliocentric orbit of the particle in a trivial manner[2], by way of equations (2) - (5). The angle α of the U-vector with the y-axis, defined by

$$\cos\alpha = U_y/U \qquad (12)$$

plays an important role in determining the heliocentric orbit of the particle.

For $U < \sqrt{2} - 1$, the largest aphelion distance attainable through encounters with one planet corresponds to $\alpha = 0^{\circ}$ and is

$$Q'_{max} = [A(1+e)]_{max} = (1+U)^2/(1 - 2U-U^2) \qquad (13)$$

For U equal or exceeding the limit parabolic and hyperbolic orbits become possible.

For $U < 1$, the smallest possible perihelion distance is different from zero; it corresponds to $\alpha = 180^{\circ}$ and is

$$Q_{min} = [A(1-e)]_{min} = (1-U)^2/(1 + 2U-U^2) \qquad (14)$$

For U equal or exceeding unity the case $Q = 0$ (falling into the sun) is allowed, although its probability is small (cf. Table 4).

By using these formulae, the probabilities of angular deflection and orbital change in a single encounter can be calculated from Equations (6) and (8).

The largest deflection, γ_{max}, corresponds to grazing passage, $r = R$. Table 1 contains some sample values for this case.

Table 1. Maximum Single Angular Deflection.

Planet	R	$2\mu/R$	U=0.1	U=0.2	U=0.5 γ_{max}	U=1.0	U=2.0
Earth	4.26×10^{-5}	0.1423	122.°5	79.°6	25.°6	7.°6	2.°0
Mars	1.48×10^{-5}	0.0438	86.7	41.5	9.3	2.5	0.6
Jupiter	8.90×10^{-5}	21.5	175.1	170.1	155.5	132.4	95.6

For comets the U-values are usually between 1 and 2. It can be seen from the table that the terrestrial planets can but slightly change the orbit of a passing comet in a single encounter.

However, for objects in short-period orbits such as periodic comets, the angular deflections accumulate according to the rule of random walk, i.e. they are summing up quadratically for successive encounters. An accumulated average deflection of 90°, or a "full deflection" can be assumed to be equivalent to the establishment of a random orientation of the U vector, without "memory" of its former direction so that the probability Θ of its being directed into solid angle ω is then

$$\Theta = \omega/4\pi \tag{15}$$

For repeated passages within the limits of from R to D the target radius σ for full deflection, to be used with equation (6) by setting $s = \sigma$, is then defined by[1]

$$\sigma^2 = B \; ln \left[(D^2 + B)/(R^2 + B)\right] \tag{16}$$

where

$$B = \left(\frac{4\mu}{\pi U^2}\right)^2 \tag{17}$$

The probability $p\Theta$ (per revolution) or $P\Theta$ (per year) of a given orbital change can then be calculated, when the solid angle ω for the changed group of orbits is given. This can be computed by numerical integration in a somewhat involved manner, except in the case of ejection from the solar system, when the parabolic limit $A \rightarrow \infty$ yields directly

$$\Theta_\infty = {}^1/_2\,(1 - \cos\alpha_\infty) = (U^2 + 2U - 1)/4U \tag{18}$$

This follows from the expression for the heliocentric velocity, W,

$$W^2 = 1 + U^2 + 2U \cos\alpha \tag{19}$$

which defines α_∞ when $W^2 = 2$ is set as for escape.

In the general case of orbital change resulting in perihelion distances less than $q = Q < 1$, or aphelion distances greater than $q = Q' > 1$, an approximate procedure is sufficient for most purposes. Defining

$$\theta_q = {}^1/_2(1 \mp \cos \alpha_q) \tag{20}$$

where the upper (-) sign corresponds to $q > 1$ (large aphelion), the lower sign to $q < 1$ (small perihelion), the effective value of $\cos \alpha_q$ can be assumed approximately equal to

$$\cos \alpha_q = {}^1/_2 (\cos \alpha_1 + \cos \alpha_2) \tag{21}$$

Here α_1 is the limiting angle which leads to an orbit with the prescribed value of q when the U-vector is in the tangential plane ($U_x = 0$), and α_2 when the U-vector is in the radial plane ($U_z = 0$). These angles are given by

$$\cos \alpha_1 = \left[(q - 1)/(q + 1) - U^2\right] /2U \tag{22}$$

for $q \gtrless 1$, and

$$\cos \alpha_2 = \left\{ (q^2 - 1) \mp \left[(q^2-1)^2 - (q-1)^2 + q^2U^2\right]^{1/2} \right\}/U \tag{23}$$

with the sign rule as in equation (20).

Special cases, when the procedure as described above does not apply, are as follows:

(1) For $q > 1$ and large U, when $\cos \alpha_1 < -1$, $\cos \alpha_2 > -1$, set in equation (21) $\cos \alpha_1 = -1$; when also $\cos \alpha_2 < -1$, set $\theta_q = 1$.

(2) For $q > 1$ and small U (q near 1), when $\cos \alpha_1 > 1$, set $\theta_q = 0$; when $\cos \alpha_1 < 1$, $\cos \alpha_2 > 1$, set in equation (21) $\cos \alpha_2 = 1$.

(3) For $q < 1$ when $\cos \alpha_1 < -1$ and both solutions α' and α'' of equation (23) are real,

$$\theta_q = 1 - \cos {}^1/_2 (\alpha' - \alpha'') \tag{20a}$$

can be assumed.

(d) Partial crossings

When the perihelion or aphelion of the particle is placed between the perihelion and aphelion of the planet, the crossing is partial. The overlapping fraction, to be used with equation (6), is then[2]

$$f = (\arc\cos E)/\pi \tag{24}$$

where

$$E = \left[A(1-e) - (1-e_o^2)\right] / \left[e_o A(1-e)\right] \tag{25}$$

for crossings near perihelion of the particle, and

$$E = \left[(1-e_o^2) - A(1+e)\right] / \left[e_o A(1+e)\right] \tag{26}$$

for crossings near aphelion of the particle.

All formulae apply without modification for $1 > f > 0.5$. However, when $f < 0.5$, U_x becomes imaginary and further adaptation of the formulae is required. Evidently, the probabilities of encounter will not vary much with small variations in the orbital dimensions of the planet. Hence, for

$$0.5 > f > 0 \tag{27}$$

in equations (2) - (5) it is proper to use $A/(1+e_o)$ instead of A for perihelion crossings, and $A/(1-e_o)$ for aphelion crossings.

When

$A(1-e) > 1+e_o$, or $A(1+e) < 1-e_o$, $|E| > 1$, f is imaginary and crossing is formally not possible. Collisions are then not allowed. However, especially with the giant planets, close encounters may still be efficient in producing angular deflection. In such a case it is advisable to use equations (2) - (5) with $A/(1+e_o+D)$ instead of A for perihelion appulses (no longer crossings), and with $A/(1-e_o-D)$ for aphelion appulses, to substitute $A(1-e)-(1+e_o)$ or $(1-e_o)-A(1+e)$ for Δ in equation (16), and subtract D from the numerators of equations (25) and (26).

Angular deflection makes ultimately real crossing and collisions possible whose second-stage probabilities can be treated by the standard formulae.

The procedure which is here described, although not precise, will yield approximations which are close enough for practical use.

(e) Auxiliary tables

Table 2 contains cross section data calculated for the principal planets, to be used for assessing the probabilities of collision and orbital change. The radius and mass of the particle are assumed to be zero, and the calculations have been made according to equations (8), (15), (16), (17), and (18). With equations (6) and (9), the interpolated value of $s^2 = S^2$ from the table yields then the probability of physical collision, and with $s^2 = \sigma^2 = S^2 \times (\sigma^2/S^2)$ the same equations yield the probability of a full deflection in angle. The table contains also the relative probabilities (Θ) of ejection and deflection to crossings with other planets.

Deflection to a close passage by the sun is also of special interest, from the standpoint of survival of the particle. The limiting condition for such a deflection is summarized in the second line of Table 3. The table lists the extreme limits of heliocentric distance attainable through repeated encounters with a single planet as calculated from equations (13) and (14).

Table 2. Collision and Angular Deflection Parameters for Close Encounters

R = radius of planet, S = target radius for physical collision, σ = target radius for full deflection, D = radius of sphere of action, all in units of the planet's mean heliocentric distance; μ = mass of the planet, in solar units; θ = relative probability of orbital change ($\rightarrow \infty$, excape to infinity; $\rightarrow$ planet, to crossing with another planet)

	Mercury			Venus			Earth		
	$R=4.18\times10^{-5}$; $D=4.38\times10^{-3}$; $\mu = 1.67 \times 10^{-7}$			$R=5.63\times10^{-5}$; $D = 0.0107$; $\mu = 2.45 \times 10^{-6}$			$R=4.26\times10^{-5}$; $D=0.01148$; $\mu = 3.03 \times 10^{-6}$		
U	S^2 (10^{-8})	σ^2/S^2	$\theta\rightarrow$ (Jupiter)	S^2 (10^{-8})	σ^2/S^2	$\theta\rightarrow$ (Jupiter)	S^2 (10^{-8})	σ^2/S^2	$\theta\rightarrow$ (Jupiter)
0.10	3.15	12.2	-	30.7	21.4	-	27.7	35.3	-
0.15	2.37	3.32	-	15.3	10.1	-	13.3	17.6	-
0.20	2.10	1.21	-	10.1	5.32	-	8.28	9.87	-
0.25	1.98	0.534	-	7.56	3.05	-	5.96	6.07	-
0.30	1.90	0.269	-	6.22	1.85	-	4.68	3.90	0.019
0.35	1.87	0.148	-	5.40	1.17	0.056	3.92	2.57	0.111
0.40	1.84	0.089	0.065	4.87	0.771	0.135	3.48	1.76	0.183
0.4142	1.83	0.077	0.086	4.78	0.696	0.155	3.32	1.59	0.200
0.50	1.80	0.037	0.197	4.27	0.368	0.254	2.85	0.889	0.292
0.60	1.78	0.018	0.294	3.82	0.195	0.340	2.54	0.489	0.372
0.80	1.77	0.006	0.433	3.60	0.067	0.468	2.22	0.179	0.491
1.00	1.77	0.0024	0.536	3.45	0.029	0.564	2.07	0.079	0.666
1.50	1.75	0.0005	0.733	3.29	0.006	0.751	1.92	0.017	0.764
2.00	1.75	0.0002	0.893	3.23	0.002	0.907	1.89	0.005	0.916
2.4142	1.75	0.0001	1.000	3.22	0.001	1.000	1.86	0.003	1.000

Table 2. Continued

U	Mars S^2 (10^{-10})	σ^2/S^2	$\theta \rightarrow$ (Jupiter)	$\theta \rightarrow$ (Earth)	Juipter S^2 (10^{-8})	σ^2/S^2	$\theta \rightarrow$ (∞)	$\theta \rightarrow$ (Earth)	$\theta \rightarrow$ (Saturn)
	$R = 1.48 \times 10^{-5}$; $D = 5.46 \times 10^{-3}$ $\mu = 3.24 \times 10^{-7}$				$R = 8.90 \times 10^{-5}$; $D = 0.0782$; $\mu = 9.55 \times 10^{-4}$				
0.10	11.8	13.3	-	-	1700	298	-	-	-
0.15	6.43	5.38	-	0.135	757	432	-	-	0.077
0.20	4.57	2.52	-	0.225	425	438	-	-	0.224
0.25	3.72	1.30	0.033	0.279	273	391	-	-	0.308
0.30	3.26	0.730	0.134	0.314	189	340	-	-	0.369
0.35	2.98	0.432	0.209	0.340	140	290	-	-	0.418
0.40	2.78	0.276	0.269	0.357	107	250	-	-	0.458
0.4142	2.75	0.245	0.283	0.361	100	241	0.000	-	0.467
0.50	2.58	0.122	0.360	0.381	69.0	189	0.125	0.052	0.522
0.60	2.47	0.061	0.429	0.395	48.0	148	0.233	0.106	0.574
0.80	2.34	0.021	0.534	0.408	27.4	96.2	0.388	0.164	0.658
1.00	2.28	0.009	0.618	0.406	17.8	67.7	0.500	0.189	0.727
1.50	2.23	0.0015	0.786	0.381	8.36	33.6	0.708	0.195	0.864
2.00	2.21	0.0006	0.934	...	5.06	19.4	0.875	...	0.974
2.4142	2.21	0.0003	1.000	...	3.71	13.1	1.000	...	1.000

Table 2. Continued

	Saturn				Uranus					
	$R = 4.07 \times 10^{-5}$; $D = 0.0522$; $\mu = 2.86 \times 10^{-4}$				$R = 8.66 \times 10^{-6}$; $D = 0.0279$; $\mu = 4.37 \times 10^{-5}$					
U	S^2 (10^{-8})	σ^2/S^2	$\theta \rightarrow$ (∞)	$\theta \rightarrow$ (Jupiter)	S^2 (10^{-9})	σ^2/S^2	$\theta \rightarrow$ (∞)	$\theta \rightarrow$ (Saturn)	$\theta \rightarrow$ (Jupiter)	$\theta \rightarrow$ (Neptune)
0.10	234	675	-	-	75.8	1330	-	-	-	-
0.15	104	652	-	...	34.7	878	-	-	-	0.172
0.20	58.8	530	-	0.129	19.0	608	-	0.040	-	0.285
0.25	37.7	423	-	0.199	12.2	446	-	0.124	-	0.360
0.30	26.2	340	-	0.244	8.47	343	-	0.180	-	0.415
0.35	19.3	278	-	0.276	6.25	270	-	0.220	0.016	0.458
0.40	14.8	233	-	0.300	4.81	219	-	0.248	0.065	0.495
0.4142	13.9	222	0.000	0.306	4.48	208	0.000	0.255	0.077	0.504
0.50	9.53	169	0.125	0.330	3.10	153	0.125	0.285	0.132	0.562
0.60	6.67	128	0.233	0.348	2.18	113	0.233	0.307	0.174	0.605
0.80	3.83	80.2	0.388	0.364	1.26	68.3	0.388	0.328	0.217	0.684
1.00	2.51	55.2	0.500	0.366	0.833	45.3	0.500	0.333	0.234	0.749
1.50	1.20	23.2	0.708	0.346	0.411	20.0	0.708	0.316	0.231	0.878
2.00	0.750	14.2	0.875	...	0.264	10.5	0.875	...	...	0.980
2.4142	0.568	9.2	1.000	...	0.204	6.55	1.000	...	...	1.000

Table 2. Continued

Neptune

$R = 5.89 \times 10^{-6}$; $D = 0.0294$; $\mu = 5.08 \times 10^{-5}$

U	S^2 (10^{-8})	σ^2/S^2	$\theta \to$ (∞)	$\theta \to$ (Uranus)	$\theta \to$ (Saturn)	$\theta \to$ (Jupiter)
0.10	59.7	2160	-	-	-	-
0.15	26.6	1450	-	0.100	-	-
0.20	15.0	1010	-	0.204	-	-
0.25	9.57	745	-	0.262	-	-
0.30	6.66	577	-	0.300	0.000	-
0.35	4.89	457	-	0.326	0.058	-
0.40	3.78	369	-	0.346	0.102	-
0.4142	3.50	352	0.000	0.350	0.114	-
0.50	2.42	260	0.125	0.371	0.164	0.041
0.60	1.69	193	0.233	0.386	0.200	0.095
0.80	0.964	119	0.388	0.398	0.239	0.156
1.00	0.631	80.0	0.500	0.398	0.253	0.182
1.50	0.300	37.2	0.708	0.407	0.384	0.190
2.00	0.184	20.6	0.875	...	...	...
2.4142	0.137	13.4	1.000	...	...	...

Table 3. Minimum and Maximum Attainable Heliocentric Distances from Close Encounters with one Planet

U	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.4142	0.50	0.60	0.80	≥1.00
Q_{min}	1.000	0.822	0.681	0.566	0.471	0.382	0.325	0.268	0.220	0.207	0.143	0.087	0.020	0.000
Q'_{max}	1.000	1.228	1.531	1.952	2.571	3.572	5.452	10.27	49.00	∞	∞	∞	∞	∞

For small values of Q, instead of equation (14), the approximation

$$Q_{min} = {}^1/_2\,(1 - U)^2 \tag{28}$$

closely holds.

Table 4 contains the probability factors as defined by equation (15) and calculated from equation (20a) for deflection to a perihelion distance of $Q_{min} \leq 0.0054$. With respect to Jupiter, this corresponds to a distance of 0.028 astron units or six solar radii, where the equilibrium black-body temperature of a sphere is 1670° K, likely to lead to rapid destruction of small solid bodies.

Table 4. Relative Probability of Deflection to Perihelion Distance 0.0054 and of Ejection to Infinity

U	≤0.896	0.9	1.0	1.2	1.5	>1.8
Θ(0.0054)	0	0.0012	0.0270	0.0134	0.0042	Hyperbolic only
Θ(∞)	≤0.445	0.447	0.500	0.592	0.708	...

According to Table 4, the probabilities of ejection, and their range of U (> 0.4142), are so very much greater than those of deflection to a small heliocentric distance, that few objects in an existing population can be expected to have undergone excessive heating in close approaches to the sun. This has an important bearing on the interpretation of the life history of meteorites.

3. DYNAMICAL PROBABILITIES OF ELIMINATION FOR COMETS AND RELATED OBJECTS

(a) Multiple crossings

The total probability of an event, depending on crossings with several planets, is conventionally assumed to be equal to the sum of the probabilities for each separate crossing. The interaction of the crossings, in making possible orbital changes which are not allowed by single crossings, is provisionally not taken into account for reasons put forward in section 2.a. This interaction, or "playing ball" with the crossing particle, can be considered as a second step[2,1]. Deflection to a crossing with a much larger planet (Jupiter) is practically equivalent to elimination of the particle from the original population, because of the much greater probability of encounter and shorter lifetime relative to the large planet.

If P_k [equation (9)] is the probability for one crossing of an event leading to the termination of existence of the particle (either destruction or elimination from the original environment), the total probability or inverse lifetime is given by

$$1/\tau = P_o = \sum P_k \tag{29}$$

and the partial probability of one particular event (collision with a planet, ejection to infinity, deflection to a crossing with a larger planet) is defined as

$$J_k = P_k/P_o \tag{30}$$

This is a true probability, conforming to its mathematical definition, whereas the P-values as used here are actually mathematical expectations.

(b) Dynamical probabilities of survival for selected lists of objects

Tables 5, 6, and 7 contain the results of calculations for typical comets and some groups of asteroids which partly (Apollo group) may be related to comets (cf. next section). In the tables, U is the relative velocity in a crossing, in units of the mean orbital velocity of the planet, calculated according to equation (5), j_c is the relative probability of collision, j_∞ (Table 5) the probability of ejection to infinity, j_J (Table 6) of deflection to Jupiter's crossing (equivalent to elimination from the terrestrial group), j_E (Table 7) of deflection to Earth's crossing (equivalent to elimination from Mars group and transfer to terrestrial group), all as defined by equations (30) and (29). τ is the lifetime, defining the fraction χ_t of the original population surviving after an interval t as

$$\chi_t = \exp. (- t/\tau) \tag{31}$$

The survival probabilities for all crossings combined are

$$J_c = \Sigma j_c, \quad J_\infty = \Sigma j_\infty, \quad J_J = \quad j_J \tag{32}$$

denoting the total probabilities of collision, of ejection to infinity, and of deflection to Jupiter's crossing, respectively. In Table 7, only j_E is given; the relative probability of physical collision with Mars is then

$$j_c = 1 - j_E \tag{33}$$

The diameters of asteroids and of the nucleus of Comet 1949 III (where the magnitude of the nucleus was observed) are calculated on the assumption of a lunar albedo[2], according to equation (40). For a few comets which have their integrated magnitude derived by a standard procedure, the few diameters of the nuclei are tentatively derived from equation (50) with C = 2.18; these diameters are marked with the letter c.

The tables are grouped according to the type of orbital elements and crossings, as these alone determine the dynamical survival without regard to the physical properties of the bodies (except when they are very small and influenced by radiation pressure and drag).

Table 5 contains only objects crossing Jupiter's orbit. The condition turns out to be equivalent to a selection of comets, and of meteor streams related to comets. The majority of periodic comets could have been incorporated in the table but for a few cases, completely listed in Tables 6 and 9. The only apparently non-cometary object of this qualification is the asteroid Hidalgo; however, there are no reliable indications as to its physical nature; it could well be an inactive cometary nucleus, rather than a runaway asteroid. As compared with a former publication where only physical collisions were considered[2], the total probabilities of elimination of the objects of Table 5 are much greater and the lifetimes shorter, as a consequence of elimination by angular deflection in Jupiter's gravitational field.

On the other hand, for the objects of Table 6 crossing only the orbits of the terrestrial planets the angular deflection is relatively insignificant, especially at high velocities, so that physical collisions dominate the process of elimination. This is even more true of the objects of Table 7 in single crossings with Mars.

It is significant that the condition of selection by Jupiter crossing in Table 5 yields numerous comets but only one doubtful asteroid Hidalgo. This is explained by the short lifetime of asteroidal objects (i.e. those of moderate inclination and eccentricity) in Jupiter crossings (cf. the second and fourth entries of Table 5), so that they are eliminated much faster than their rate of injection into the Jupiter group. On the other hand, the rate of injection into the group os comets by capture from the non-periodic complex[3] is very much higher than from the asteroidal population, so that many of the captured comets are still observable and prevail in the list, despite their rapid disappearance.

Table 5. Dynamical Probabilities of Elimination for Selected Objects in Crossings with Jupiter

Object	944 Hidalgo	Comet 1939 IV Väisälä	Comet 1942 II Väisälä	Comet Giacobini-Zinner (Giacobinids)	Comet 1866 I Tempel (Leonids)	Comet 1862 III Tuttle (Perseids)	Comet Halley (η Aquarids, Orionids)
Diam. nucleus, Km	45.	...	...	1.1 c	...	13. c	11. c
Period, yrs.	13.9	10.52	85.5	6.59	33.2	119.6	76.0
a(1-e), astron. un.	1.995	1.752	1.287	0.995	0.977	0.963	0.587
a(1+e), astron. un.	9.59	7.85	37.53	6.02	19.67	47.60	35.31
i, degrees	41.0	11.3	38.0	30.8	162.7	113.6	162.2
				Venus crossing			
U	-	-	-	-	-	-	2.32
J_c	-	-	-	-	-	-	0.006
J_∞	-	-	-	-	-	-	0.0000
				Earth Crossing			
U	-	-	-	0.694	2.35	2.01	2.23
J_c	-	-	-	0.001	0.009	0.007	0.003
J_∞	-	-	-	0.0003	0.0000	0.0000	0.0000
				Jupiter crossing			
U	0.966	0.677	1.281	0.729	1.91	1.81	1.90
J_c	0.021	0.026	0.031	0.027	0.044	0.043	0.044
J_∞	0.711	0.974	0.893	0.972	0.805	0.844	0.816
				Saturn Crossing			
U	0.747	-	1.305	-	1.71	1.72	1.77
J_c	0.008	-	0.005	-	0.008	0.006	0.007
J_∞	0.260	-	0.060	-	0.119	0.091	0.104
				Uranus crossing			
U	-	-	1.201	-	1.32	1.57	1.55
J_c	-	-	0.0002	-	0.001	0.0003	0.0006
J_∞	-	-	0.004	-	0.015	0.003	0.006
				Neptune crossing			
U	-	-	1.073	-	-	1.40	1.30
J_c	-	-	0.0002	-	-	0.0003	0.0006
J_∞	-	-	0.008	-	-	0.006	0.012
				All crossings			
J_c	0.029	0.026	0.036	0.028	0.062	0.057	0.061
J_∞	0.971	0.974	0.964	0.972	0.938	0.943	0.939
τ, 10^8 yrs	2.27	0.340	37.	0.504	14.0	162.	39.

Table 6. Dynamical Probabilities of Elimination for Objects in Multiple Crossings with Terrestrial Planets (Complete List)

Object	Encke's Comet (Taurids)	Comet 1949 III Wilson Harrington	Geminids	Apollo	Adonis	Hermes
d, Km	1.7 c	5.9	...	1.0	1.3	0.4
a, astron, un.	2.22	1.746	1.38	1.49	1.97	1.29
a(1-e)	0.338	1.0276	0.140	0.65	0.44	0.68
a(1+e)	4.10	2.47	2.62	2.34	3.51	1.90
i, degrees	12	2.2	24	6	1.5	5
			Mercury Crossing			
U	0.581	-	1.10	-	0.459	-
J_c	0.27	-	0.23	-	0.02	-
J_J	0.002	-	0.000	-	0.0001	-
			Venus Crossing			
U	0.924	-	1.17	0.400	0.754	0.319
J_c	0.42	-	0.44	0.58	0.40	0.61
J_J	0.009	-	0.007	0.060	0.017	0.000
			Earth Crossing			
U	1.000	0.219	1.16	0.574	0.856	0.485
J_c	0.25	0.772	0.28	0.26	0.48	0.29
J_J	0.013	0.000	0.011	0.052	0.040	0.072
			Mars Crossing			
U	1.029	0.430	1.07	0.597	0.896	0.455
J_c	0.03	0.214	0.04	0.04	0.04	0.03
J_J	0.000	0.014	0.000	0.001	0.0003	0.002
			All Crossings			
J_c	0.975	0.986	0.983	0.884	0.943	0.926
J_J	0.025	0.014	0.017	0.116	0.057	0.074
τ, 10^6 yrs	265.	330.	245.	64.	68.	39.

Table 6. <u>Continued</u>

Object	Icarus	1950 DA	Geographos 1951 RA	1948 OA	1948 EA
d, Km	1.4	1.3	2.8	4.8	6.3
a, astron. un.	1.08	1.65	1.24	1.38	2.26
a(1-e)	0.19	0.84	0.83	0.77	0.89
a(1+e)	1.98	2.46	1.65	1.98	3.63
i, degrees	23	12	13	10	18
		Mercury Crossing			
U	0.979	-	-	-	-
J_c	0.22	-	-	-	-
J_J	0.0003	-	-	-	-
		Venus Crossing			
U	1.040	-	-	-	-
J_c	0.44	-	-	-	-
J_J	0.009	-	-	-	-
		Earth Crossing			
U	1.004	0.449	0.382	0.443	0.696
J_c	0.28	0.73	0.71	0.72	0.81
J_J	0.014	0.21	0.22	0.22	0.11
		Mars Crossing			
U	0.827	0.556	0.342	0.462	0.809
J_c	0.04	0.06	0.07	0.06	0.08
J_J	0.0004	0.002	0.006	0.004	0.0008
		All Crossings			
J_c	0.976	0.787	0.781	0.784	0.890
J_J	0.024	0.213	0.219	0.216	0.110
τ, 10^8 yrs	165.	272.	152.	185.	1010.

Table 7. Dynamical Probabilities of Elimination for Objects in Single Crossings with Mars (complete list)

Object		d Km	a(1+e)	U	τ 10^8 yrs	J_E	Object		d Km	a(1+e)	U	τ 10^8 yrs	J_E
132	Aethra	89	3.61	0.512	24.5	0.042	1204	Renzia	13	2.93	0.205	3.17	0.353
323	Brucia	54	3.10	0.470	22.0	0.058	1221	Amor	1.8	2.76	0.483	5.31	0.053
391	Ingeborg	30	3.03	0.447	15.4	0.068	1235	Schorria	6.8	2.21	0.486	10.5	0.054
433	Eros	20	1.78	0.289	1.84	0.205	1293	Sonja	12	2.83	0.190	3.24	0.374
475	Ocllo	35	3.58	0.444	20.5	0.071	1310	Villigeria	28	3.24	0.487	17.5	0.051
699	Hela	23	3.68	0.392	17.9	0.097	1316	Kasan	11	3.18	0.460	17.3	0.064
719	Albert	5.2	3.98	0.502	7.56	0.045	1374	Isora	11	2.87	0.221	4.83	0.333
887	Alinda	6.2	3.89	0.510	6.62	0.043	1468	Zomba	11	2.86	0.281	8.52	0.214
985	Rosina	14	2.98	0.210	3.68	0.353	1474	Beira	26	4.07	0.619	15.8	0.022
1009	Sirene	5.2	3.82	0.437	10.8	0.074	1508	1938 UO	10	3.93	0.609	29.8	0.022
1011	Laodamia	10	3.23	0.266	4.48	0.242	1580	Betulia	4.4	3.44	1.006	17.8	0.003
1036	Ganymed	59	4.10	0.661	16.4	0.019	...	1950 LA	3.5	2.28	0.578	7.55	0.028
1131	Porzia	8.7	2.93	0.200	2.87	0.362	...	1951 SA	40	4.02	0.415	10.7	0.083
1134	Kepler	6.6	3.94	0.442	11.1	0.068	...	1953 EA	0.6	3.85	0.666	14.2	0.019
1139	Atami	12	2.44	0.298	4.03	0.190	...	1953 RA	8	3.26	0.452	6.85	0.067
1170	Siva	22	3.02	0.479	18.8	0.055	...	1951 QZ	11	3.08	0.232	2.14	0.326
1198	Atlantis	5.2	3.00	0.228	1.84	0.316	...	1957 NA 1929 SH	5.5	2.59	0.421	4.02	0.078

Table 8. Dynamical Probabilities of Elimination of Comets in Partial Crossings with Jupiter, $0 < f < 0.5$, $4.94 \leq a(1+e) \leq 5.20$ (Complete List)
τ = total lifetime; τ_σ = time of 90° deflection in angle

Object	Period, yrs	a(1+e)	a(1-e)	i deg.	U	τ 10^6 yrs	τ_σ 10^3 yrs	J_c	J_∞
DeVico-Swift	5.86	5.11	1.39	3.0	0.386	0.93	3.4	1.00	0.00
Holmes	6.86	5.10	2.12	20.8	0.408	5.5	22.	1.00	0.00
Whipple	7.41	5.15	2.45	10.2	0.272	1.5	4.1	1.00	0.00

Table 9. Dynamical Probabilities of Elimination of Comets in Close Appulses to Jupiter's Perihelion, $4.94 \geq a(1+e) \geq 4.54$ (complete list) (cf. Table 8 for notations)

Object	Period, yrs	a(1+e)	a(1-e)	i deg.	U	τ 10^6 yrs	τ_σ 10^3 yrs	J_c	J_∞
Grigg-Skjellerup†	4.90	4.92	0.85	17.6	0.587	0.21	50.	0.02	0.98
Tempel (2)	5.31	4.70	1.39	12.4	0.404	4.4	80.	1.00	0.00
Neujmin (2)	5.43	4.84	1.34	10.6	0.442	0.63	30.	0.11	0.89
Tempel (1)	5.98	4.82	1.77	9.8	0.367	3.5	16.	1.00	0.00
Schwassmann-Wachmann	6.53	4.83	2.15	3.7	0.318	1.2	3.6	1.00	0.00
Oterma (1934)	18.0	8.07	5.65	2.9	0.148				
" (1950)	7.9	4.54	3.40	4.0	0.100	1.6*	0.8*	1.00	0.00
" (1965)	19.2	8.99	5.35	1.9	0.145				

* Averaged over the three orbits.

† Diameter of nucleus = 0.4 Km from equation (50).

Table 10. Would-be* Dynamical Probabilities of Elimination of Asteroids in Close Appulses to Jupiter's Perihelion, $4.94 > a(1+e) > 4.54$. Representative samples (cf. Table 8 for notations)

Object	a(1+e)	i	U	τ 10^6 yrs	τ_σ 10^3 yrs	J_c	J_∞
153 Hilda	4.59	8.7	0.179	0.68	1.5	1.00	0.00
499 Venusia	4.84	3.9	0.195	0.92	2.2	1.00	0.00
525 Adelaide	4.58	2.1	0.221	0.22	0.6	1.00	0.00
1038 Tuckia	4.88	8.4	0.249	2.7	10.	1.00	0.00

* The close appulses, however, apparently cannot take place, on account of near commensurability of the periods of planet and Jupiter and ensuing preventive perturbations, as in the case of Pluto with respect to Neptune[1], so that the lifetimes are extended indefinitely.

Table 11. Complete List of Asteroids in Would-be Close Appulses to Jupiter's Perihelion, $a(1+e) \geq 4.54$. n/n_J = ratio of mean motion to Jupiter's; n'/n'_J ratio of angular motion in aphelion to that of Jupiter in perihelion; L_a = longitude of aphelion; L_c = longitude of conjunction nearest to epoch; 12° = longitude of Jupiter's perihelion; ΔL_c = displacement in longitude per conjunction; T_{ac} = nearest epoch of conjunctions in aphelion.

Object	d Km	a	a(1+e)	n/n_J	n'/n'_J	$L_a - 12^\circ$	$L_c - 12^\circ$	ΔL_c	T_{ac} A.D.
153 Hilda	150	3.975	4.59	1.497	1.018	-143°	-70°	+4°	1520
190 Ismene	200	3.947	4.61	1.513	1.004	89	74	-18	1930
361 Bononia	110	3.936	4.77	1.519	0.959	-19	106	-26	2070
499 Venusia	125	3.963	4.84	1.503	0.915	-11	18	-4	2110
525 Adelaide	58	3.340	4.58	1.944	0.936	90	-63	+21	1990
748 Simeisa	98	3.934	4.65	1.520	0.990	-7	132	-28	2070
958 Asplinda	68	3.934	4.66	1.520	0.960	-32	60	-28	2030
1038 Tuckia	49	3.917	4.88	1.530	0.891	115	-12	-41	1870
1180 Rita	74	3.988	4.69	1.486	0.910	24	-124	+21	2110
1202 Marina	59	3.930	4.74	1.522	0.948	123	-46	-30	1820
1212 Francette	55	3.965	4.68	1.502	0.973	162	170	-3	2020
1345 Potomac	76	3.968	4.66	1.501	0.984	47	96	-1	3100
1512 1939 FE	38	3.955	4.63	1.508	1.000	55	-112	-10	1550
1529 1938 BC	32	3.996	4.76	1.485	0.945	111	-7	+22	2060
1578 Kirkwood	17	3.959	4.84	1.506	0.915	174	50	-9	1610

(c) Dynamical survival of comets in incomplete or near crossing with Jupiter

Tables 8, 9, and 10 contain data for objects in partial crossing or in near appulses to Jupiter, calculated with the aid of the rules and equations of section 2(d). In the case of Jupiter, the two-body approximation of close encounters is no longer good; it becomes worse for imperfect crossings. Nevertheless, the calculated lifetimes for a given set of orbital elements still may be reliable to within 30-50 per cent. The objects of these tables are especially important in evaluating possible relationships between comets and asteroids; for this purpose the knowledge of the survival time scales to a close order of magnitude is quite sufficient.

For the objects of Table 9, collisions with Jupiter are not allowed immediately, but become possible as a consequence of orbital change through angular deflection; the probabilities of collision were calculated therefore as a second step. In view of orbital change, these objects are only temporarily inside Jupiter's perihelion; about 70 per cent of their lifetime they are expected to be in crossing with Jupiter. From this standpoint, there is little difference between the objects of Table 8 and Table 9. However, Comets De Vico-Swift and Tempel (1) have periods in near commensurability of 1/2 with Jupiter's and may be relatively stable, as are 525 Adelaide and other asteroids discussed in the following subsection (d). In such a case the calculated lifetimes and probabilities do not apply to these two objects.

The elements of Comet Oterma (Table 9) are changing in a very peculiar manner during close approaches to Jupiter [4]; of the three calculated orbits, only one (1965) is a close appulse according to our conventional definition; only for this a calculation was made and the probability divided by 3, to obtain a kind of average for all three calculated orbits. The case is on the borderline of application of our schematically defined probabilities.

(d) Asteroids in close appulses to Jupiter

These are selected by their aphelia being potentially within the sphere-of-action distance from Jupiter's perihelion; a complete list is given in Table 11, with the omission of Hidalgo which belongs to a different group (Table 5).

Fourteen out of fifteen entries of Table 11 belong to the Hilda family of asteroids, with the periods of revolution in a near commensurability ratio of 2/3 to that of Jupiter (cf. 5th column of the table); the only exception is 525 Adelaide, with a period ratio close to 1/2.

The lifetimes of representative objects of this group, calculated by the conventional methods, are given in Table 10. Although at present none of these objects trespasses over Jupiter's perihelion of 4.94, it is conceivable that but a slight deflection in angle of the U-vector would bring them into real crossing with Jupiter, when physical collision becomes possible. The probability of collision, p_c, is then calculated as a second step from the probability of angular deflection, p_σ,

$$p_c = f\theta_J p_\sigma S^2/\sigma^2$$

in former notations, with $\theta_J = 0.70$ very closely and $f = 0.5$ as for half-crossing.

The lifetimes (5th column of Table 10), of the order of one million years, are surprisingly short as compared with the time scale of the solar system. The objects would have been eliminated in the very beginning of the solar system if the calculated probabilities were valid.

In addition, within time intervals of the order of τ_σ (6th column of Table 10), or a few thousand years, the exclusive distribution of the aphelia would be upset completely, about 70 per cent being expected to reach beyond 5.20 a.u., Jupiter's mean distance. This certainly is not the case, and the absence of larger aphelia can in no way be ascribed to observational selection.

Eighty per cent of the objects of Table 11 exceed in diameter (actually brightness) the lone exception, Hidalgo (Table 5); if the latter were an escaped asteroid, larger and more easily observable objects in similar orbits should exist, which apparently is not the case. Besides, the lifetime of Hidalgo as an object in two-fold crossing with Jupiter and Saturn is definitely short and cannot refer to a case of "long storage", whatever the commensurability ratio of the present period with Jupiter's[1].

We conclude that, by some interplay of perturbations in nearly commensurable periods, the asteroids of Table 11 are not only virtually stable, but even their aphelia are somehow made to comply with the limit of Jupiter's perihelion. The mechanism preventing close approaches of these objects to Jupiter may be similar to that of Pluto with respect to Neptune[1], although more complicated owing to the greater relative extent of the sphere of action of Jupiter, as compared with Neptune's. The mechanism is directly related to close commensurability of the periods[1].

OrtodoX methods of celestial mechanics seem to fail in the case of these objects. Thus, if Chebotarev's calculations [5] of secular perturbations of the asteroids of the Hilda family are taken at their face value, we would be made to believe that the present statistical picture of their orbital elements, including near commensurabilities of the mean motions and the limitation of the aphelia to less than 4.94 a.u., is a rare coincidence, valid only for our time $\pm$ 100 years. A few hundred years before or after, the mean motions and aphelia, according to Chebotarev, would have been spreading over a wide range of values, without any trace left of the present statistical regularity. The probability of the present peculiar distribution to have taken place accidentally is less than $(\frac{1}{3})^{15}$ for the mean motions, less than $(\frac{1}{2})^{15}$ for the aphelia, or a total probability of less than 10^{-12}.

it is not reasonable to accept such an improbable coincidence. Clearly, Chebotarev's calculations of secular perturbations of the planets of the Hilda family made by expanding into exponential and trigonometric series of time as the only variable, cannot be valid over time intervals exceeding 100 years (1).

A method of calculation which would give realistic results consists in numerical integrations of the space motion of these objects, and not of abstractions such as orbital elements. The relative accuracy of the integrations need not be excessively high; a nominal accuracy of 10^{-8} would suffice. If automatic regulation of the orbits and close approaches exists, it will reveal itself in the calculations despite errors. In other words, the mechanism of regulation will equally respond to true imperfections (in the orbital elements, as well as to spurious imperfections) caused by the method of calculation. A spurious divergence of mean longitude and other rotating elements with time may result , but the (a,e,i) set of elements must keep within definite limits in spite of errors of calculation.

4. GENETIC RELATIONSHIPS OF COMETS AND ASTEROIDS

(a) Origin of comets and structure of nuclei

It is almost impossible to conceive how the cloud of comets, situated at 5×10^4 to 1.5×10^5 astron. units from the sun[3], could have come there into being by condensation of diffuse matter; under any reasonable assumptions as to the original mass (1 solar mass) and density of the solar nebula (less than 10^{-22} gr/cm^3), and with the low molecular velocities at these distances and low temperatures, of the order of 10^4 cm/sec for the component normal to the accreting surface (the orbital velocity (10^3 cm/sec) being negligible), the maximum size of solid particles accreted in four billion years could have been of the order of 0.1 cm, and much less during the first few hundred million years of the formation of the solar system. The formation of comet nuclei 1-100 Km in diameter is out of the question under these circumstances. They must have originated in much denser regions of space, closer to the sun.

From this standpoint, a suggestion by Oort[3] that the comets are "minor planets escaped, at an early stage of the planetary system, from the ring of asteroids, and brought into large, stable orbits through the perturbing actions of Jupiter and the stars", deserves particular attention as the only consistent hypothesis of the origin of comets. In addition to Jupiter, it may be that primordial rings of asteroids from the vicinity of other planets may also have contributed to the cloud of comets. The formulae and tables of the two preceding sections can be used to describe quantitatively this process of escape, as well as the subsequent capture of these object by Jupiter and the other planets into periodic orbits, when the mixing action of stellar perturbations[3][6] happens to bring them back into the inner portions of the solar system. This, however, is not the purpose of the present investigation.

The common origin would imply some common properties in the physical structure of comets and asteroids. We do not know much about the physical structure of these bodies, except that asteroids must contain compact solid substance similar to that of meteorites, whereas comets must carry on their surfaces a mixture of ices and dust (Whipple's mixture[7]) which is not obviously present in the asteroids. If Oort's concept is correct, it is likely that the asteroidal fragments have acquired a coating of the icy conglomerate in the very beginning, when the temperature in their region was very low — the sun's radiation being screened off by intervening dust. Those fragments which were sent away to the distant regions of the solar system have retained their icy coating; they appear thus as comet nuclei, although their cores may be similar to the asteroids. The latter have lost their ices by evaporation, at least from near the surface, after the dust had cleared out of the inner portions of the solar system and solar radiation became effective.

The mechanical structure of comet nuclei still remains a mystery. In all probability, very different types of structure may exist. The old theory of a cluster of particles bound together by mutual gravitation and easily disrupted by tidal action near the sun, is still upheld in some recent papers, although the role of collisions in leading to condensation of the cluster is not overlooked[8]. Undoubtedly, small particles cannot account for the persistence of the gaseous emissions from comets over a great number of revolutions. These emissions must originate in bulky objects. Also, well-known cases of comets splitting up into two or more components of a similar order of magnitude indicate that the number of components is small, and their sizes comparable. It is inadmissible to assume that the comets have split into several independent clusters. The centers of these division products must be bulky objects, not clusters which would have completely dispersed in the process of fission.

It appears to be plausible to assume that comets are not composed of clusters of small particles crossing each other's orbits; these must have been eliminated very soon in mutual collisions. What has survived of the original structure may be single or multiple bulky nuclei, orbiting in a regular manner without crossing, similar to the principal planets of the solar system or to multiple star systems built on the hierarchical principle, i.e. with the orbital dimensions of successive members increasing by orders of magnitude (a close binary with a distant companion, etc.). Loss of mass by evaporation and tidal action may then lead to the observed fission.

Evaporation of the ices from a bulk nucleus may leave behind a giant dustball structure which, at a density of $\delta = 0.6$ gr/cm^3 and a minimum strength of the order of $s = 10^4$ dyne/cm^2, assumed equal to that observed in cometary dustball meteors[9][10], will withstand compression at the center from own gravity (central pressure equals $^1/_8 \pi G \delta d^2$, where G = gravitational constant) up to a diameter of d = 7 Km.

With a conductivity as low as that of lunar dust, it can be shown that an icy conglomerate sphere of 2 Km diameter may take 3×10^8 years to lose all its ices by evaporation if the dust is not removed from the surface. Exhausted or "dead" comet nuclei may thus exist as dustballs up to this limit of size, comparable to the members of the Apollo group (Table 6); beyond this size they may still possess an icy core surrounded by uncompacted dust layers.

The dustball structure of dead comet nuclei and, perhaps, of some asteroids may account also for a peculiar object, Comet Wilson-Harrington 1949 III (see Table 6) which entirely belongs to the Mars - Earth space and is well separated from Jupiter's perihelion. The gravitational field of Mars is too weak to have achieved its capture from the outside field with some probability; the relative velocity, $U = 0.430$, is high enough to permit an origin from Jupiter's crossing, but the time scale for capture from the Jupiter field, $\tau/j_J = 2.4 \times 10^{10}$ years, is rather long as compared with the total lifetime. On the other hand, interaction with the earth has a short enough time scale, but, on account of the low relative velocity, $U = 0.219$, encounters with earth cannot send this object to Jupiter's crossing (cf. Table 3), nor could an object captured by the earth from Jupiter's field have a velocity less than $U = 0.29$. The object is thus almost completely isolated in terrestrial space.

As to the calculation of lifetime, Comet 1949 III does not cross the present orbit of the earth. The approach is, however, so close that variations in the orbital eccentricity of the earth would make crossing possible even if the orbit of the object itself did not change. From calculated eccentricities over ± 400,000 years[11] it follows that during 32 per cent of the time the eccentricity of the earth's orbit exceeded $e_{\odot} > 0.0276$ and had then a mean value of 0.0346. This gives $f = 0.32f'$, with $f' = 0.21$ according to equations (24) and (25). The probability of collision with earth was then calculated according to equation (6).

It is thus improbable that Comet Wilson-Harrington was ever captured into terrestrial space from outside; this object may be an asteroid of the icy conglomerate type, with ices in the interior preserved and insulated by an outer dust layer. The impact of another asteroid or meteorite may have thrown open the interior, exposing the ices to direct heating and evaporation, with the ensuing cometary appearance. (paragr.) A dustball structure of 2.2 km diameter can resist tidal disruption at grazing passage by the earth[12][13] by virtue of its cohesion; for compact ice the limit of diameter is 50 — 70 km ; beyond a distance of four earth radii from earth, or 2.5 solar radii from sun (Roche's limit), dusball structures of any size will be held together by their own gravitation. Hence it is clear that even loosely bound compact nuclei can exist indefinitely, and that the observed breakup of some comets into separate nuclei only can be explained by these nuclei being separated beforehand, orbiting around the common center of gravity at distances which are much greater (100 times, to name a figure) than the diameters of the nuclei themselves. Some comet nuclei, at least, must consist of multiple gravitating systems with a small number of principal members; others may be single bulk bodies.

(b) Types and physical survival of comet nuclei

In Table 6, there are listed eight known objects of the Apollo group, coexistent with and of similar orbital properties and dynamical age as the three cometary entries of the table (Encke's comet, the Germinids and Comet 1949 III). There is some reason to suspect that their physical origin may also be similar, partly at least.

A cometary nucleus, or a proto-asteroid which, according to Oort's concepts, has become a comet, may have a structure of one of the following two basic types: type I, entirely consisting of Whipple's icy-dust conglomerate[7]; and type II, partly consisting of solid meteorite chunks or even one solid nucleus, surrounded by the icy conglomerate. When becoming a periodic comet, the icy conglomerate partly evaporates in the sunlight, partly scatters as dust and dustball meteors, a process usually called disintegration. Type I either may disintegrate completely, leaving only meteoric matter dispersed in space behind, or may survive as a giant dust-ball. Type II, after losing its volatile and dust coating, will become almost unobservable, but its solid meteoritic portion will continue in the orbit. In both cases the residual nuclei will appear as asteroids similar to the objects of the Apollo group, only being observable under favorable circumstances.

Whether the Apollo group is likely to contain such nuclei of dead or "disintegrated" comets, can be decided statistically with the aid of the probabilities of elimination. The answer turns out to be in the positive. The apparent disintegration time of periodic comets is estimated to run into about 70 revolutions[3] or 10^3 years; although the figure is rather uncertain, it is sufficient to show that the rate of disintegration is certainly very much faster than that of dynamical elimination (time scale 10^6-10^8 years). Therefore, the number of dead nuclei of comets must exceed the number of live periodic comets of type II by many orders of magnitude. If all were of type II, the number of asteroids in cometary orbits would probably greatly exceed the actual number, as can be judged from the lists in Tables 6 and 5 (Apollo group and Hidalgo). Although a definite estimate cannot be made, from the relative scarcity of Apollo type objects we have to conclude that most comet nuclei are of type I which disintegrate completely and that only a few are of type II, or of type I leading to a residual giant dustball.

The simultaneous existence of cometary nuclei of both types may be understood on the following working hypothesis. At an early stage in the inner portions of the solar nebula the temperature must have remained at a low level, on account of absorption by dust along the ecliptical plane; there the first objects to condense were planetesimals of the icy conglomerate type I. Some of them were ejected into the present cometary cloud. Others further agglomerated, forming sizable planets in which the conglomerate was ultimately differentiated into gas and solid rock. Before the nebula cleared, collisions were breaking up some of the planets, releasing asteroidal and meteoritic compact fragments which, after covering up with the hoar-frost of the icy mixture, were also partly ejected to the cometary cloud, forming thus nuclei of type II. Being of later origin, these may be expected to be less numerous than the nuclei of type I, and the composition of their conglomerates may also be different; the diversity of comet spectra and tails may be understood on these lines as a difference in early age.

(c) <u>Dead comet nuclei and the origin of the Apollo group</u>

Excluding the three cometary objects of Table 6, comet Encke, the Germinids, as well as Comet Wilson-Harrington, the harmonic mean lifetime of the eight apparently asteroidal members of the Apollo group is found to equal

$$\tau_A = \left[(1/\tau)_{av}\right]^{-1} = 1.02 \times 10^8 \text{ years} \qquad (34)$$

If these were the remnants of a population <u>in situ</u> which has decreased exponentially according to equation (31), the original number 4500 million years ago would have been 2.5×10^{19} times greater and, allowing for the considerable incompleteness of the list due to observational limitations, would correspond to a total of 100 times the sun's mass. The absurdness of such an assumption is obvious. In any case, the survival since the beginning of the solar system of the three shortlived objects, Apollo, Adonis, and Hermes, would be ~~more~~ as probable ~~than~~ as a miracle.

We have to conclude therefore that the asteroids of the Apollo group are not permanent members of the space occupied by the terrestrial planets where they are now but, while they are eliminated chiefly by collisions on a time scale of 10^8 years, they are currently supplied from some source or sources, so that the balance of the population is maintained. Two sources can be thought of: the asteroidal belt, and comets.

None of the aphelia of the regular asteroids reaches to Jupiter's perihelion (Table 11), so that there is no crossing with the giant planet; this is readily explained by rapid elimination of crossing objects[2]. And, as has been pointed out in Section 3.(d), the present asteroids of the Hilda family must be virtually stable, otherwise they would have disappeared long ago. Also, none of the aphelia [a(1+e), Table 6] of the eight members of the Apollo group comes anywhere near Jupiter's orbit, the larges values being 3.51 and 3.63 a.u., well below those of the outer asteroids of Table 11. An origin from the asteroidal belt by way of Jupiter's perturbations seems thus to be excluded.

Next come the asteroids crossing Mars, an almost complete up to date list of which is given in Table 7. As can be seen from the τ-values, these are long-lived objects which well could have been present there since the origin of the solar system. The harmonic mean lifetime for the 34 asteroids listed is

$$\bar{\tau}_M = 6.02 \times 10^9 \text{ years} \tag{35}$$

and longer than the age of the solar system. According to equation (31), in 4500 million years their numbers must have decreased by 50 per cent, so that about one-half of the original population may have survived in Mars crossings. Unlike the other terrestrial planets, the interaction cross section of Mars is small enough to make reasonably probable a prolonged coexistence with crossing asteroids.

Although collisions with Mars are the chief source of removal of these objects, a not negligible fraction (j_E, Table 7) is diverted to Earth crossings and is thus injected into the Apollo group. From Table 7, the average value of j_E weighted by $1/\tau$ is

$$(j_E)_{av} = 0.211 \tag{36}$$

and the annual injection rate from the Mars asteroids (Table 7) into the Apollo group (Table 6) becomes

$$I_{ME} = 0.211\ N_M/\tau_M = 3.50 \times 10^{-11}\ N_M \tag{37}$$

when the numerical value of τ_M is substituted from equation (35); here N_M is the population of the Mars group. The annual loss from the Apollo group, as corresponding to the lifetime defined by equation (34), is

$$L_A = 9.8 \times 10^{-9}\ N_A \tag{38}$$

where N_A is the population of the Apollo group.

Let L_{AM}, N_{AM} in equation (38) refer to that part of the population of the Apollo group which derives from the Mars group. If the limits of selection (e.g. by diameter) are the same and statistical equilibrium holds

$$L_{AM} = I_{ME}$$

whence, from equations (38) and (37),

$$N_{AM} = 0.0036\ N_M \tag{39}$$

Now, the selection limits of the two lists are not comparable and are strongly depending on diameter. The Apollo group list goes down to a diameter of about 1.0 km, whereas the Mars group is equally complete (or incomplete) to about 5 km. We may attempt an evaluation of the selection effects.

For the Mars group we may use statistical data on apparent magnitudes of asteroids in general. It has been found concordantly by different authors (Stroobant, Baade, Putilin, Öpik) that the increment in cumulative numbers (i.e. total sum down to a certain diameter) of asteroids in the observed range of size proceeds nearly with the -1.6 power of the limiting diameter (population index = 1.6)[14]. By extrapolation of the numbers with the aid of this index (which requires a trebling of the number for a decrease to one-half of the diameter) and basing on the number of large diameters assumed to be listed completely, the data of Table 12 are obtained.

Table 12. Selection by Size in Mars Group of Asteroids

Limits of diam., km	>68	34.- -68.	17.- -34.	8.5- -17.0	4.2- -8.5	2.1- -4.2	1.05- -2.1	0.52- -1.05
Number in list, first half	1	3	4	4	5	0	0	0
Number in list, second half	0	1	2	7	4	1	1	1
Total number in list	1	4	6	11	9	1	1	1
Observed cumulative number	1	5	11	22	31	32	33	34
Extrapolated true cumulative number	1	5	15	45	135	405	1215	(3645)

Taking $N_M = 1215$ for $d > 1.05$ km, according to the last line of Table 12, equation (39) yields an equilibrium population of the Apollo group, $N_{AM} = 4.4$, as maintained by injection from the Mars asteroids.

The number listed in Table 6 with $d > 1.0$ km is 7 which would seem to be close enough to the expectation. Actually, however, the list of the Apollo group cannot be complete; there are undoubtedly many more undiscovered objects in the group. The selection effects are difficult to allow for, but their order of magnitude may be estimated as follows. The present list of the Apollo group is mainly due to charting with the 48-inch Mount Palomar Schmidt. For the sake of simplicity we assume that the sky has been efficiently covered twice with this instrument to a limiting magnitude 18.5 (for a moving object at 1.0 a.u. geocentric) distance. This probably exaggerates the completeness of coverage. With lunar albedo, the conventional diameter (d in km) of an object of apparent magnitude m in mean opposition is

$$\log d = 3.63 + \log r\Delta - 0.2\, m \qquad (40)$$

where r and Δ are heliocentric and geocentric distance in a.u., respectively[2].

Allowing for the effect of motion on the photographic plate, the photographic image intensity varies as Δ^{-1}, not as Δ^{-2} with geocentric distance, and the limiting minimum diameter of an asteroid, observable with a given instrumental set, thus varies with the two distances as

$$d_{min} \sim r\,\Delta^{1/2} \tag{41}$$

Setting $r = 1$, $\Delta = 1$, $m = 18.5$ in equation (41), $d_{min} = 0.8$ km obtains at the unit distances for opposition; under an average phase angle of $30°$ this somewhat reduces perhaps to $d_{min} = 1.2$ km as an effective limit. Hence, according to equation (41),

$$d_{min} = 1.2\, r\Delta^{1/2} \tag{41a}$$

in km, for r and Δ in a.u.

On account of phase effects, the discovery of the fainter asteroids can take place mainly when they are outside the earth's orbit, $r > 1$, $\bar{r} = 1 + 0.5\Delta$, whence

$$d_{min} = 1.2\,(1 + 0.5\Delta)\,\Delta^{1/2} \tag{42}$$

If V_0 is the total volume of space effectively occupied by the asteroids, and V is the volume covered by observation, the "coefficient of perception" or the relative completeness of the resulting list of discoveries is

$$\eta = 1 - \exp\,(-V/V_0) \tag{43}$$

For asteroids of the Apollo group moving between effective distances of $q = 0.7$ and $q' = 2.0$ a.u., the volume occupied is approximately

$$V_0 = \tfrac{4}{3}\pi\left[(q')^3 - q^3\right] = 32\ (\text{a.u.})^3 \tag{44}$$

For an observing distance Δ and double coverage of the sky

$$V = \tfrac{8}{3}\pi\Delta^3 = 8\Delta^3 \tag{45}$$

Hence

$$\eta = 1 - \exp(-\tfrac{1}{4}\Delta^3) \tag{46}$$

and this is linked to the minimum diameter by way of equation (42). Hence follows the coefficient of perception of the Apollo list depending on diameter as given in Table 13.

Table 13

Δ, a.u.	0.1	0.2	0.3	0.4	0.6	0.8	1.0	1.5	2.0
η	2.5×10^{-4}	0.0020	0.0068	0.016	0.054	0.13	0.22	0.57	0.86
d_{min}, km	0.40	0.59	0.76	0.91	1.15	1.50	1.80	2.57	3.39

Statistics of the small number of Apollo asteroids cannot be very significant; nevertheless, with population indices of s = 2.7 (probable value[14]) and 1.6 (asteroidal), the idealized distribution of the objects is found to be as in Table 14.

Table 14.

Limits of d, km	1.00--1.19	1.19--1.41	1.41--1.68	1.68--2.00	2.00--2.38	2.38-2.83	>2.83	All >1.00
η	0.04	0.08	0.15	0.25	0.35	0.60	0.80	
Population index s = 2.7								
n	10.2	6.4	4.1	2.6	1.6	1.0	1.6	27.5
ηn	0.41	0.51	0.62	0.65	0.56	0.60	1.28	4.63
Population index s = 1.6								
n	4.00	3.00	2.28	1.72	1.32	1.00	3.20	16.52
ηn	0.16	0.24	0.34	0.43	0.46	0.60	2.56	4.79
ηn observed	1	3	0	0	0	1	2	7

Here n is the idealized true relative number of objects (to an arbitrary factor of proportionality), η the average coefficient of perception according to Table 13, ηn the expected number of observed objects.

For a population index of s = 2.7, d $\gtrsim$ 1.0 km, the probable ratio of the true number to the number of discovered objects is 27.5/4.63 = 6.1;

for $s = 1.6$, it is $16.52/4.79 = 3.5$. The case $s = 2.7$ better agrees with the observed distribution of diameters. It means that the 7 objects of the Apollo group with $d > 1.0$ km are representing a true population of about $N_A = 7 \times 6.1 = 43$. The estimate errs probably on the lower side. The fact that each of the asteroids of the Apollo group has been observed only once, by mere chance at discovery, and then hopelessly lost, would indicate that only a very small fraction of them is presently known.

In any case, the probable number N_A as estimated here is ten times the number N_{AM} expected from the injection of Mars asteroids; the injection is apparently inadequate.

Hence it appears that the majority of the Apollo asteroids cannot have originated from the asteroidal belt. They may indeed be dead comet nuclei, or other objects infiltrating from the region of the cloud of comets, such as true asteroidal bodies not covered with the hoar-frost coating and thus unobservable except at close distance.

Asteroidal collisions as a source of Apollo type fragments might be considered next. Leaving aside the infrequency of such collisions, and the circumstance that the aphelia of the Apollo group are not crowded toward the densest portion of the asteroidal belt, but are spreading definitely inwards of it, there is one argument which makes the suggestion unacceptable. To reach the earth after collision from a distance of 2.7 - 3.1 a.u., the fragments must acquire a relative velocity in our notation of $U \cong 0.3$ or 5 km/sec. The average velocity of collision of two asteroids is of the order of

$$U = (2 \sin^2 i + 2e^2)^{1/2} \cong 0.28$$

or barely equal to the required velocity of ejection of the fragments. This is definitely inadequate, considering that most of the kinetic energy of the collision is released inelastically. The fragments can attain only a fraction of the velocity of collision. Moreover, from the theory of meteorite crater formation(15)(16) it appears that in a collision with cosmic velocity, large fragments cannot survive the accelerations required to eject them with large velocities. Only small meteoritic fragments can arrive to us from the asteroidal region as the result of collisions, not bodies measuring kilometers or hundreds of meters in diameter, even when consisting of a material as hard as compact nickel iron. The idea must be abandoned with respect to the Apollo group.

5. CAPTURE OF COMETS INTO TERRESTRIAL SPACE AND STATISTICAL BALANCE WITH APOLLO GROUP.

(a) Definition

Terrestrial space is here defined as that inside Jupiter's orbit. The orbital characteristics of objects belonging to this space are defined by the absence of Jupiter crossings, or by the aphelia being less than 4.94 a.u. However, the range from 4.54 to 4.94 a.u. remains within the sphere of action of Jupiter as a transition region. The objects of Tables 6 and 7 belong to this space, although Table 7 represents a more narrow selection of Mars space. In particular, the Apollo group is characteristic of terrestrial space.

(b) Statistical equilibrium of disintegration and dynamical elimination

Let N_c be the number of comets in terrestrial space, τ_c the lifetime of their "disintegration", i.e. evaporation of the volatile substances with all its consequences, and k_2 the fraction of Type II or other surviving nuclei among them; and let N_{AC} be the number of objects in the Apollo group derived from the "Disintegration" of the comets, τ_A their dynamical lifetime. Equilibrium conditions require

$$k_2 N_c / \tau_c = N_{AC} / \tau_A$$

or

$$k_2 N_c = N_{AC} \tau_c / \tau_A \tag{47}$$

Assuming $N_{AC} = N_A - N_{MC} = 43-4 = 39$ for $d \gtrsim 1.0$ km, according to section 4(c), $\tau_A = 10^8$ years, $\tau_c = 10^4$ years which is more than usually ascribed to comets, we obtain $k_2 N_c = 0.004$ as the required time-average number of "live" comets in the terrestrial space, capable of yielding residual nuclei exceeding 1.0 km in diameter. In Table 6, there is one bona fide comet of $d = 1.7$ km * and a meteor stream (Geminids) which must have recently (on a time scale of 10^8 years) been formed from a disintegrated comet. In addition, periodic comet Grigg-Skjellerup has its aphelion exactly at the limit of 4.94 a.u. (Table 9). Only objects with Apollo group characteristics, i.e. crossing the orbit of the earth and potentially capable of reaching Jupiter are thus included. The observed number may thus be set at $N_c = 1.5$ ($d > 1.0$ km); this is satisfied by a very low margin of efficiency, $k_2 = 0.003$. The estimate is extremely uncertain, but sufficient to show that the hypothesis of some members of the Apollo group having been derived from disintegration of comets does not require many live comets to be present in the terrestrial space.

(c) Injection from general field of comets

From the "new" comets temporarily entering terrestrial space from outer regions, the terrestrial planets may partly eliminate some by collisions, partly they may perturb their motions in close encounters, capturing them into terrestrial space. If the outer planets were not there, the probabilities of collision and capture could be calculated from equation (6), with

* As suggested in section 4(a), Comet Wilson-Harrington is more properly counted with the bona fide asteroids deriving from the Mars region.

the encounter target radii being given by equations (8) and (17) and the relative probability of a certain orbital change being defined by equation (15). In such a case the relative probability for the changed orbit to be entirely inside Jupiter's orbit is evidently

$$\theta_i = 1 - [\theta \rightarrow (\text{Jupiter})] \tag{48}$$

with $\theta \rightarrow$ (Jupiter) being given in Table 2.

In the presence of the outer planets, the collision probabilities will not be affected. However, orbital change of crossing orbits is so efficiently caused by Jupiter that the angular deflections induced by the terrestrial planets cannot accumulate in random walk and equation (17) does not apply. Only deflections in single encounters are effective. These impose severe limitations on the possibility of orbital change.

The calculation of probabilities of orbital capture in single encounters is rather complicated, except for parabolic objects. For them, the writer estimated by numerical integrations [using equation (11)] that, from an isotropically distributed population of parabolic objects [$W^2 = 2$, equation (19)], captures by the earth can take place only in the velocity range of $U = 0.48 - 0.72$, and the total probability of capture into terrestrial space per crossing and perihelion passage of a parabolic comet ($\sqrt{2}-1 \leq U \leq \sqrt{2}+1$) is then

$$P_A = 1.0 \times 10^{-11}$$

The captures take place in a close range of perigee passages between 1.00 and 1.09 earth radii.

Application of the cumulative random walk procedure with equations (17) and (15) to 25 nearly parabolic or long-period (>90 years) objects which have been observed crossing the earth's orbit during 1936 - 1949[17] yielded an average of

$$P'_A = 2.9 \times 10^{-11}$$

The average is sensitive to individual values of p which flutuate considerably. Thus, excluding comet 1941 II, which has the largest value of the probability, the average becomes

$$P'_A = 0.6 \times 10^{-11} \quad .$$

One can see that, although theoretically the cumulative procedure is not justified, in practice it yields a numerical result sufficiently close to the correct one for deflection in single encounters.

It can be assumed that the cumulative procedure yields an acceptable approximation also in the case of the periodic orbits, where the exact calculation is complicated, but which chiefly contribute to the probability as being more easily captured than the parabolic objects.

From a complete list of comets over 14 years(17), for 30 observed apparitions of objects crossing the orbits of earth and Jupiter (repeated apparitions of periodic comets being counted individually), the average probabilities per apparition of collision (P_c) and of capture (P_A) into Apollo-type orbits of terrestrial space, were found as listed in Table 15. In notations of Table 2, in each individual case

$$P_A = P_c(\sigma^2/S^2) \quad [1 - \theta \rightarrow (\text{Jupiter})] \quad .$$

Table 15

	By Earth	By Venus	Total per Apparition
Probability of collision, P_c	45.9×10^{-10}	13.8×10^{-10}	59.7×10^{-10}
Probability of capture, P_A	2.65×10^{-10}	0.27×10^{-10}	2.92×10^{-10}

If ν_c is the true number of apparitions per year of comets whose nuclei are of the right size, to become ultimately members of a population N_{AC} of the Apollo group with an efficiency factor k_2, for statistical equilibrium

$$P_A k_2 \nu_C = N_{AC}/\tau_A \tag{49}$$

or, with $\tau_A = 10^8$, $P_A = 2.9 \times 10^{-10}$, $N_{AC} = 39$ ($d > 1.0$ km),

$$k_2 \nu_C = 13 \times 10^3 ;$$

for $1 > k_2 > 0.003$, this is equivalent to from 10^3 to 4×10^5 comets per year with nuclei exceeding 1 km in diameter crossing the orbit of the earth; such a number is absolutely out of the question.

This statistical puzzle can also be treated in a more direct way. If the observational selectivity of comets in general and those in terrestrial space is the same, instead of guessing the very uncertain selection factors, the adequacy of injection can be tested directly from the number of recorded objects. In the preceding subsection it has been shown that one or two live comets, actually known to be present in terrestrial space, are amply sufficient for maintaining the population of the Apollo group at its present level, even with an efficiency as low as 0.003 for the fraction of surviving residual nuclei. It remains thus to account for the origin of the observed number of live comets in terrestrial space, $N_C = 1.5$. In notations of this and the preceding subsections, we have then

$$N_C = \nu_o P_A \tau_C \qquad (47a)$$

where ν_o is the annual number of all observed apparitions without selection effects. With $\nu_o = 30/14 = 2.1$, $P_A = 2.9 \times 10^{-10}$, $\tau_C = 10^4$ years, $N_C = 6 \times 10^{-6}$ obtained, which is utterly insignificant as compared with an observed effective number of 1.5. According to this criterion too, direct capture of field comets by the earth cannot account for adequate injection into terrestrial space, the rate being short by a factor of 10^5.

(d) Comet diameters and magnitudes

If the evaporation intensity per unit surface of the nucleus, and thus the total brightness of a comet, is a unique function of heliocentric distance, the diameter of a comet nucleus must be given by a formula of the type

$$\log d = C - 0.2\, m_o \qquad (50)$$

where m_o is the Bobrovnikoff-Schmidt "absolute" or standard magnitude reduced to unit heliocentric and geocentric distances and to standard instrument[18][19]. The equation may involve considerable deviations in individual cases, but should apply as an average.

For the period 1853-1948, of almost a century, for which photometric data are available, the absolute magnitude distribution and number of apparitions of bright comets, with perihelion distances less than 1.02 a.u., is given in section (a) of Table 16, according to Vanysek[20]; in section (b) the statistics of all apparitions is presented according to the catalogue of Baldet and Obaldia[21]

Table 16. Absolute Magnitudes and Apparitions of Bright Comets ($q \leq 1.02$ a.u.)

(a)

Relative diameter	>6.3	4.0	2.5	1.6	1	0.62	0.40
m_o, mag.	≤1.9	2.0-2.9	3.0-3.9	4.0-4.9	5.0-5.9	6.0-6.9	7.0-7.9
Number, 1853-1948	1	0	2	5	13	17	5
Cumulative number, N	1	1	3	8	21	38	43
Population index, s		...	2.4	2.1	2.1	(1.3)	...

(b)

Period of observation	1853 - 1899	1900 - 1935	1936 - 1948
(I) Total apparitions, $q \leq 1.02$	121	72	33
(II) Number with $m_o \leq 5.9$	11	7	3
Selection Ratio (I) to (II)	11.0	10.3	11.0
Apparitions per year	2.58	2.00	2.35

The first line in section (a) of the table gives the relative diameter in units of that of the $m_o = 5.0 - 5.9$ group, according to equation (50). The population index of diameters[14], defined here as

$$s = 5\ d(\log N)/dm_0 \ , \tag{51}$$

is given in the fifth line of the table. Its decrease beyond $m_0 = 5.9$, and the statistical comparison with section (b) of the table indicates that down to $m_0 = 5.9$ there is no relative selection in the photometric data of these bright comets whence, down to this limit, the average population index of diameters is $\bar{s} = 2.1$. With this, the cumulative numbers increase in a ratio of 4.3 for a ratio of limiting diameters of one-half, or in a ratio of 2.63 per magnitude interval of m_0.

Allowing for unfavorable perihelion passages, we may estimate that, despite their relative completeness as compared with the total number of apparitions, probably only one-quarter of all comets with $m_0 \leq 5.9$ have been recorded, so that the true annual number of comets brighter than the sixth magnitude crossing the orbit of the earth may be estimated at

$$\nu_6 = 21 \times 4/96 = 0.88.$$

With $s = 2.1$ the true number of apparitions down to a magnitude limit $m = m_0 > 6$ can be extrapolated with the aid of the formula

$$\nu_m = 0.88 \times 2.63^{(m-6)}$$

For lunar albedo, equation (40) yields for the magnitude m_c of the nucleus at unit distances ($r = 1$, $\Delta = 1$, or the same for which the integrated magnitude m_0 of the comet is calculated) and full phase

$$m_c = 18.15 - 5 \log d_c \tag{52}$$

whence from equation (50)

$$m_c - m_0 = 18.15 - 5C \tag{53}$$

The virtual invisibility of the true nuclei of most comets would imply $m_c - m_0 > 5$ or $C < 2.6$ as an overall upper limit for the average value of the parameter.

A direct determination of the constant C is possible only on rare occasions. From two cases when the true nucleus was observed, the writer made a reduction of the photometric observations to the conventional 3-inch telescope standard and unit heliocentric distance using an exponent $\bar{n} = 3.3$ for the heliocentric reduction (Bobrovnikoff-Schmidt[18][19] reduction). Assuming lunar albedo, the results for the constant in equation (50) were: $C = 2.25$ for periodic Comet Harrington-Abell 1955a; $C = 2.15$ for Comet 1946a Timmers. An average of $C = 2.18$ was actually adopted and used[14][15]. If the albedo of the nuclei is as low as that of zodiacal dust, which is not improbable for the dust-covered radiation-damaged surface, $C = 2.6$ would be indicated. Undoubtedly, individual comets may differ widely in this respect, according to the surface rate of evaporation of the ices, but the average may be used for statistical purposes.

As shown in Table 17, the "minimum" diameters of comet nuclei proposed by Richter[22] are by almost an order of magnitude greater than our larger set of values ($C = 2.6$).

Table 17.

Comet	1903 IV	1904 I	1907 IV	1932 V	1932 VI	1936 II	1937 IV
m_o	6.5	3.4	4.3	7.4	5.1	6.8	6.2
d [equation (50), $C=2.6$], km	20	83	55	13	38	17	23
d [equation (50), $C=2.18$], km	7.6	32	21	5.0	13	6.6	8.7
d_{min} (Richter), km	307	232	194	73	200	106	77
m_C (Richter diameter, lunar albedo)	5.7	6.3	6.7	8.8	6.6	8.0	8.7

The values of m_C, calculated from Richter's diameters, require the nuclei to yield from 7 to 200 per cent of the total light of the comet at unit heliocentric distance. From the observational standpoint this is an unacceptably high, partly impossible ratio. There can be little doubt that

Richter's diameters are too large for compact nuclei, and that most of the light he attributed to the nucleus must have come from the "false nucleus", the concentration of gas and dust leaving the nucleus in all directions.

Even the values calculated with $C = 2.6$ appear to be too high. The number of craters in the lunar Mare Imbrium, essentially depending on the frequency and size distribution of comet nuclei, and calculated with $C = 2.18$ for the comets, agrees with the observed number[14]. The agreement disappears when $C = 2.6$ is assumed, increasing the volume and mass of each projectile 18 times. The calculated number of craters for given size limits would then increase about 5 times and exceed in this ratio the observed number. Whatever the uncertainties in the estimate of the frequency of lunar craters, it would be quite difficult to bridge over this gap.

For working purposes, and with a tentative probable error, we thus can assume for the parameter of equation (50)

$$C = 2.18 \pm 0.2 \quad ,$$

or values between 2.0 and 2.4 as the extreme range.

There exists an independent check on this figure which carries more weight than any estimates of the diameters of comet nuclei ever made. It is based on the apparent decrease in the gravitational constant k_2, caused by the inertial reaction of the sunward jet of vapors from the nucleus (rocket effect). Hamid and Whipple[23] have published relevant data for 64 comets with definitive orbits which yielded a significant weighted mean value of $\Delta k/k = -0.53 \times 10^{-5}$ in the expected direction. From their few most accurate entries it can be judged that the real spread in $\Delta k/k$ is of the order of $\pm 0.6 \times 10^{-5}$. Therefore, using only the very best data for which the observational mean error in $\Delta k/k$ is less than this spread, there remain six comets (1861 I, 1853 III, 1886 II, 1882 I, 1858 VI and 1896 III) for which also the standard magnitude m_0 on the Bobrovnikoff-Schmidt scale[18][19] has been determined. With a correcting factor of 0.89, assumed to represent the fraction of solar heat used up in the sublimation of the ices (lunar albedo and surface radiation loss at -100°C being assumed), and with $\Delta k/k = -0.67\ (\pm 0.18\ \text{p.e}) \times 10^{-5}$, equation (1) by Hamid and Whipple[23] yields the harmonic mean diameter for these six comets as

$$d_k = 3.2 \pm 0.8\ (\text{p.e.})\ \text{km}$$

based on the jet effect.

For the same six comets equation (50) with $C = 2.18$ yields a harmonic mean of

$d_m = 5.8$ km (extreme range from 3.6 to 9) based on the observed standard magnitudes.

The agreement is better than ever obtained in estimates of comet diameters; it seems that here at last a reliable basis has been found for assessing the true dimensions of comet nuclei.

The estimate from $\Delta k/k$ gives the mass per unit surface of the nucleus; that based on m_0 yields the total surface of the nucleus. If both estimates are taken by their face value, they could be reconciled by assuming an average comet nucleus to consist of three spherical components, each of an average diameter of 3.2 km, so that their total reflecting surface would equal that of a sphere of 5.8 km diameter. In view of what was said about the multiple structure of comet nuclei, this model and the absolute dimensions of the nuclei may be close to the truth despite the uncertainties of the estimates. The occurrence of multiple meteor craters on earth (Kaalijärv in Estonia, Henbury in Australia, and others) adds further weight to this concept.

(e) Gravitational capture from Jupiter's family of comets

This is a two-stage process, Jupiter capturing comets from the general field, the terrestrial planets, chiefly the earth capturing into terrestrial space comets with aphelia near Jupiter's orbit. The probability of capture by the earth from Jupiter's family is some three orders of magnitude greater than directly from the long-period or parabolic field whence, despite the smaller number of objects, the expectation of capture is very much greater than from the general field.

The population of the Jupiter family we consider as given, without inquiring how it got there. Capture into terrestrial space is then the net balance between incoming and outgoing objects, a problem of diffusion inwards. The capture is non-cumulative, achieved by individual deflections according to equation (11). There is a minimum value of γ which can produce the required orbital change, and this sets an upper limit to the perigee distance r and the target radius σ_{max}. For a small change in the desired direction, the probability of the change per encounter varies from nearly one-half (0.45-0.48) at grazing passage to zero at σ_{max}; an average value of the probability per target cross-section $\pi\sigma_{max}^2$ is obtained by integration.

For an original set of elements with a = 3.00 a.u., a(1-e) = 0.8, a(1+e) = 5.2, a change to typical elements of a "captured" orbit, a = 2.8, a(1-e) = 0.8, a(1+e) $\leq$ 4.8 can be achieved by earth encounters at conditions set forth in Table 18.

Table 18. Conditions for Earth Encounters to Decrease Particle's Aphelion by 0.4 a.u. from 5.2 to 4.8 a.u. or less (U > 0.29)

U	0.30	0.35	0.40	0.5	0.6	0.8	1.0	1.5	2.0
Probability of desired change at grazing passage	0.45	0.48	0.48	0.48	0.47	0.47	0.46	0.43	0.35
σ_{max}, earth radii	10.8	21.4	23.0	21.6	19.3	14.9	11.6	6.8	3.3

For the range of U from 0.3 to 0.6, as is actually covered by the relevant objects of Tables 8 and 9, the probabilities and target radii in Table 18 vary but moderately, leading to an almost constant probability of "capture" (or an almost equal probability of a change in opposite direction) of

$$p_t = 7.3 \times 10^{-8}$$

per orbital revolution; with an average period of revolution of 5 years, this defines the probability of capture per annum as

$$P_t = 1.5 \times 10^{-8}$$

"Capture" is here identified as a decrease of the aphelion distance by 0.4 a.u. or more.

From a population of N_j comets in the Jupiter family, capable of reaching earth's crossing ($U > 0.44$ with respect to Jupiter is the condition, cf. Tables 2 and 3), a fraction P_t is injected annually into a population N_c of live comets in terrestrial space. Of the latter, the same fraction P_t is returned to the Jupiter family (when $U > 0.30$ with respect to earth), and a fraction $1/\tau_c$ decays or disintegrates. The statistical equilibrium condition can then be written as

$$P_t N_j = P_t N_c + N_c/\tau_c$$

or

$$N_c = N_j/(1 + 1/P_t\tau_c) \tag{54}$$

With $P_t = 1.5 \times 10^{-8}$ yr^{-1}, $\tau_c = 10^4$ yr, this becomes

$$N_c = 1.4 \times 10^{-4} N_j \tag{55}$$

From Tables 8 and 9, $N_j = 2$ is the observed value, as only two objects (Grigg-Skjellerup and Neujmin 2) satisfy the condition $U > 0.44$. Hence

$$N_c = 3 \times 10^{-4} \tag{56}$$

is the observed equivalent value of live comets in terrestrial space, in equilibrium with the injection rate. This is very much less than the actual observed number, $N_C = 1.5$. Thus, although the equilibrium value of N_C is now 40 times that which can be sustained by injection from the general field [cf. subsection (c)], it is still unable to account for the actual number of live comets present in terrestrial space.

Of course, the statistics in this case is based on a single object (Comet Encke) and the conclusion that "there are too many object" need not be significant (however, the sampling error for one observed event may lie within the limits of -50 to +100 per cent and the result still carries definite weight).

Disregarding the single event and assuming that the average observable number of live comets in terrestrial space equals the value of equation (55) calculated from injection, in former notations the observable number (without selection being allowed for) of residual nuclei in the Apollo group becomes

$$N'_A = N_C \tau_A / \tau_C = 3,$$

or of the right order of magnitude if the efficiency, k_2, is assumed equal to unity.

Thus, the two-stage injection by capture from the Jupiter family almost can account for the population of the Apollo group except that the improbable assumption of $k_2 = 1$ must then be made. In such a case comet Encke would represent a freak whose probability to be present in a random sampling by time (over intervals of $\tau_C = 10^4$ years) is 3×10^{-4}.

(f) Capture from Jupiter's family of dead nuclei

In addition to live comets in terrestrial space, the Apollo group could be supplied by injection of extinct nuclei still crossing Jupiter's orbit. The probable ratio of the number n_J of dead nuclei to that of live comets in Jupiter crossings is

$$n_J/N_J = k_2\tau/\tau_C \tag{57}$$

in former notations, with τ = dynamial lifetime in Jupiter crossings. The harmonic mean lifetime of the nine objects of Tables 8 and 9 is $\bar{\tau} = 8.5 \times 10^5$ years; with $\tau_C = 10^4$,

$$n_J/N_J = 85\ k_2 .$$

For $k_2 < 1$, $N_J = 2$ as for objects which can cross the orbit of the earth,

$$n_J < 170 \quad ,$$

and from equation (55) the observable number of dead nuclei diverted to the Apollo group from Jupiter crossings becomes

$$n_C < 0.024$$

which again is negligible. Although the dead nuclei of comets in Jupiter's family may be numerous, Hidalgo possibly being an outstanding example, they hardly can help in understanding the origin of the Apollo population through gravitational capture.

(g) <u>Evolution through jet deceleration</u>

Whipple's ideas(7) about jet accelerations or decelerations of comets, by way of a lag in evaporation from the surface of a rotating comet nucleus, may be considered next as a possible cause of bringing members of the Jupiter family into terrestrial space. Nuclei in retrograde rotation in which a tangential jet force operates in a direction opposite to the orbital motion may lead to loss of angular momentum, chiefly near the perihelion, and to a decrease in the aphelion distance and semi-major axis. The apparent acceleration of Comet Encke may be attributed to this cause(7).

As a rough estimate, we may assume that 25 per cent of all nuclei have retrograde rotation in the plane of the orbit, with a lag of maximum evaporation of 30° in longitude (corresponding to a hour angle of 2 p.m.). With a velocity of the escaping gases equal to 0.56 km/sec, or a component $0.56 \times \pi/4 = 0.44$ km/sec in the equatorial plane, the tangential component opposing orbital motion is $0.44 \sin 30^{\circ} = 0.22$ km/sec. At unit distance from the sun (1 a.u.) this equals 0.0073 in the U-units. Hence, for a loss of mass in the ratio of M_2/M_1, the loss in the perihelion velocity becomes

$$\Delta U = - 0.0073 \ln (M_1/M_2) \tag{58}$$

The identity of the comet may still remain preserved for a mass loss ratio of $M_1/M_2 = 8$, whence

$$\Delta U = - 0.015 \tag{59}$$

appears to be the order of magnitude of the maximum attainable loss of angular momentum per unit mass. Equation (13) then yields the following variation in the aphelion distance for a perihelion at 1 a.u.:

$a(1+e)$, initial	5.7	4.95	4.00
$a(1+e)$, final ($\Delta U = -0.015$)	4.9	4.4	3.54

The orbital change is sufficient to lead to practical injection of all comets with $a(1 + e) < 5.2$ into terrestrial space. The time scale of the process is τ_C, equal to that of apparent disintegration. In notations of the preceding sections we have then (with 25 per cent of the comets subject to the effect)

$$N_C/\tau_C = {}^1/_4\ N_j/\tau_C$$

or

$$N_C = {}^1/_4\ N_j = 0.5$$

The present value of N_C is 1.5, of the same order of magnitude as that calculated.

The "rocket effect" in retrograde rotation is thus quantitatively adequate in supplying live comets to terrestrial space. Comet Encke appears to be still drifting inwards.

(h) Conclusion

The supply of live comets, suspected parent bodies of the majority of the Apollo group asteroids, is efficiently achieved by an inward drift of some of the comets of the Jupiter family, caused by the jet deceleration of nuclei in retrograde rotation. Only a small fraction of the Apollo group, of the order of 20 per cent or less, can be regarded as bona fide asteroids, diverted to earth crossing through accumulated perturbations by Mars.

The loss in U, the Jacobian relative velocity, as due to the rocket effect is comparatively small, of the order of -0.015. Hence the objects which have filtered into terrestrial space must have retained almost their original U values. This may be used as a means of guessing their possible origin in individual cases.

To have originated from the Jupiter family, $U > 0.29$ is required with respect to the earth, $U > 0.18$ with respect to Mars. Hence all objects of Table 6, with the exception of Comet Wilson-Harrington, may well have originated from the Jupiter family. The average U-value with respect to Mars of all 11 objects in this table is $\bar{U} = 0.679$, with a range from 0.382 to 1.07. For the 34 Mars asteroids of Table 7 the corresponding figures are $\bar{U} = 0.429$, range 0.190 - 1.006. There is indeed a marked difference between the two groups, suggestive of different origin; the smaller U-value for the Mars asteroids is in harmony with the hypothesis that they are indigenous to the Mars space, whereas the higher relative velocity of the Apollo group objects and related comets would indicate a more extraneous origin. As to the relatively small aphelion distances of the objects of Table 6 (except its most recent member, Comet Encke), they may have been brought about by the accumulated effect of close approaches to the terrestrial planets. The probabilities are not too favorable for this assumption, but it is difficult to find a more plausible explanation.

References

1. E. J. Öpik, Ann. Acad. Scient. Fennicae, A. 3, 1 (1961).

2. E. J. Öpik, Proceed. Roy. Irish Acad. 54 A, 165 (1951); Armagh Observatory Contribution No. 6.

3. J. H. Oort, Bull. Astron. Inst. Netherl. 11, 91 (1950).

4. Liisi Oterma, Astron.-Opt. Inst. Turku Informo No. 17 (1958).

5. G. A. Chebotarev, Bull. Inst. Theor. Astron. Leningrad 5, 393 (1953); G. A. Chebotarev and A. I. Boshkova, ibidem 5, 571 (1954).

6. E. J. Öpik, Proceed. Amer. Acad. Arts and Sc. 67, 169 (1932); Harvard Reprint No. 79.

7. F. L. Whipple, Astrophys. J. 111, 375 (1950); 113, 464 (1951).

8. E. Schatzman, Mémoirs in-8° Soc. Roy. Sc. Liège 13, 313 (1953).

9. E. J. Öpik, Irish Astron. J. 4, 84 (1956).

10. E. J. Öpik, Physics of Meteor Flight in the Atmosphere, Interscience Publ., Inc., New York (1958).

11. T. Matukuma, Sendai Astron. Raportoj 2, 115 (1949).

12. H. Jeffreys, Monthly Not. Roy. Astr. Soc. 107, 260 (1947).

13. E. Öpik, Irish Astron. J. 1, 25 (1950).

14. E. J. Öpik, Monthly Not. Roy. Astron. Soc. 120, 404 (1960).

15. E. J. Öpik, Irish Astron. J. 5, 14 (1958); Armagh Observ. Contrib. No. 24.

16. E. J. Öpik, "Notes on the Theory of Meteor Craters", Proceedings of Cratering Symposium March 28-29, 1961, Washington, D.C. (to be published).

17. J. Bouška, Acta Astronomica Sér. c 5, 33 (1953).

18. N. T. Bobrovnikoff, Contrib. Perkins Observatory No. 16 (1942).

19. M. Schmidt, Bull. Astron. Inst. Netherl. 11, 258 (1950).

20. V. Vanysek, Contrib. Astron. Inst. Masaryk University 1(9), 1 (1952).

21. F. Baldet and Miss G. de Obaldia, "Catalogue Général des Orbites de Comètes," CNRS, Paris (1952).

22. N. Richter, Astronom. Nachr. 172, 41 (1948).

23. S. E. Hamid and F. L. Whipple, Astron. J. 58, 100 (1953).